Chapter 6

Absorption and Scattering

When we have both variations in both absorption and scattering, the solution to the heterogeneous diffusion equation is a superposition of our previous solutions. The Born Solution is

$$U_{sc}(\mathbf{r}_d,\mathbf{r}_s) = - \int_V d\mathbf{r}\ \nabla U_o(\mathbf{r},\mathbf{r}_s)\cdot\nabla G(\mathbf{r}-\mathbf{r}_d)\frac{\delta D(\mathbf{r})}{D_o} \tag{6.1}$$

$$+ \int_V U_o(\mathbf{r},\mathbf{r}_s)G(\mathbf{r}-\mathbf{r}_d)\frac{v\delta\mu_a(\mathbf{r})}{D_o}, \tag{6.2}$$

and the Rytov is

$$\phi_{sc}(\mathbf{r}_d,\mathbf{r}_s) = - \frac{1}{U_o(\mathbf{r}_d,\mathbf{r}_s)}\int_V d\mathbf{r}\ \nabla U_o(\mathbf{r},\mathbf{r}_s)\cdot\nabla G(\mathbf{r}-\mathbf{r}_d)\frac{\delta D(\mathbf{r})}{D_o} \tag{6.3}$$

$$+ \frac{1}{U_o(\mathbf{r}_d,\mathbf{r}_s)}\int_V U_o(\mathbf{r},\mathbf{r}_s)G(\mathbf{r}-\mathbf{r}_d)\frac{v\delta\mu_a(\mathbf{r})}{D_o}. \tag{6.4}$$

This leads us to the following matrices to invert,

<u>Born</u>

$$\begin{pmatrix} U_{sc}(\mathbf{r}_{s1},\mathbf{r}_{d1}) \\ \vdots \\ U_{sc}(\mathbf{r}_{sm},\mathbf{r}_{dm}) \end{pmatrix} = \begin{pmatrix} W^{BA}_{11} & \ldots & W^{BA}_{1n} & W^{BS}_{11} & \ldots & W^{BS}_{1n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ W^{BA}_{m1} & \ldots & W^{BA}_{mn} & W^{BS}_{m1} & \ldots & W^{BS}_{mn} \end{pmatrix} \begin{pmatrix} \delta\mu_a(\mathbf{r}_1) \\ \vdots \\ \delta\mu_a(\mathbf{r}_n) \\ \delta D(\mathbf{r}_1) \\ \vdots \\ \delta D(\mathbf{r}_n) \end{pmatrix}$$

$$W_{ij}^{BA} = -U_o(\mathbf{r}_j, \mathbf{r}_{si}) \cdot G(\mathbf{r}_{di} - \mathbf{r}_j) v h^3 / D_o \tag{6.5}$$

$$W_{ij}^{BS} = \nabla U_o(\mathbf{r}_j, \mathbf{r}_{si}) \cdot \nabla G(\mathbf{r}_{di} - \mathbf{r}_j) v h^3 / D_o \tag{6.6}$$

Rytov

$$\begin{pmatrix} \phi_{sc}(\mathbf{r}_{s1}, \mathbf{r}_{d1}) \\ \vdots \\ \phi_{sc}(\mathbf{r}_{sm}, \mathbf{r}_{dm}) \end{pmatrix} = \begin{pmatrix} W_{11}^{RA} & \dots & W_{1n}^{RA} & W_{11}^{RS} & \dots & W_{1n}^{RS} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ W_{m1}^{RA} & \dots & W_{mn}^{RA} & W_{m1}^{RS} & \dots & W_{mn}^{RS} \end{pmatrix} \begin{pmatrix} \delta\mu_a(\mathbf{r}_1) \\ \vdots \\ \delta\mu_a(\mathbf{r}_n) \\ \delta D(\mathbf{r}_1) \\ \vdots \\ \delta D(\mathbf{r}_n) \end{pmatrix}$$

$$W_{ij}^{RA} = -U_o(\mathbf{r}_j, \mathbf{r}_{si}) \cdot G(\mathbf{r}_{di} - \mathbf{r}_j) v h^3 / (U_o(\mathbf{r}_{di}, \mathbf{r}_{sj}) D_o) \tag{6.7}$$

$$W_{ij}^{RS} = \nabla U_o(\mathbf{r}_j, \mathbf{r}_{si}) \cdot \nabla G(\mathbf{r}_{di} - \mathbf{r}_j) v h^3 / (U_o(\mathbf{r}_{di}, \mathbf{r}_{sj}) D_o) \tag{6.8}$$

We use these matrices, and the save inversion techniques (SIRT, SVD) to create separate images of absorption and scatttering.

6.1 Simulation Results

For the following simulations the forward data were generated using the exact solution for one or more spheres embedded in an otherwise homogeneous infinite medium [10]. Random noise (0.5% amplitude and 0.5^o phase) was added to all simulated data. Unless otherwise stated, the background optical properties of the medium were $\mu_a = 0.05$ cm^{-1}, $\mu_s' = 10.0$ cm^{-1} and the source modulation frequency is 500MHz. The sources and detectors rotate around a 6.0 cm diameter cylindrical region making a measurement every 20 degrees. The reconstructed area is broken up into 0.25 cm by 0.25 cm pixels and 2000 SIRT iterations were performed. The two-source subtraction technique was not used here. For simplicity, the analytic solution for the incident wave was subtracted as part of the simulation.

We have reconstructed images of scattering and absorption simultaneously using the Rytov approximation. The results of such a reconstruction are shown in figure 6.1, where 4 samples are considered.

In the first sample, there is only an absorbing inhomogeneity as verified by the reconstruction. In the second sample there is only a scattering inhomogeneity, however the reconstruction produced some noise in the absorption map. We see that an absorbing and a scattering object can be individually imaged. The third sample demonstrates that we can simultaneously image two objects, one absorbing object and one scattering object. Note in the fourth panel a single sphere with different absorption and scattering factor is imaged. We have displaced the scattering and absorption features in the figure to emphasize this point.

An important practical advantage of the Rytov approximation is its natural separation of the amplitude and phase. As stated in equation 5.45, the scattered phase is the logarithm of the ratio of the measured signals; written

$$\phi_{sc} = ln[U/U_o] = ln[Aexp(i\phi)/A_o \exp(i\phi_o)] = ln[A/A_o] + i(\phi - \phi_o). \tag{6.9}$$

Thus the amplitude decay corresponds to the real part of the integral, and the phase shift to the imaginary. Figure 6.2 shows that at 500 MHz, the amplitude ratio (i.e. amplitude with inhomogeneity relative to amplitude without inhomogeneity) and phase change for various objects with either a pure scattering change or absorption change. We see that scattering and absorption changes affect the amplitude to the same degree. However, scattering changes affect the phase a great deal more than an absorption change.

This suggests that the scattering (absorption) reconstructions might be done independently using just the phase (amplitude) of the measured wave. The results of this procedure are presented in figure 6.3. This simulation suggests that while the scattering reconstruction using only phase data is quite successful, the absorption reconstruction using only amplitude data is quite noisy.

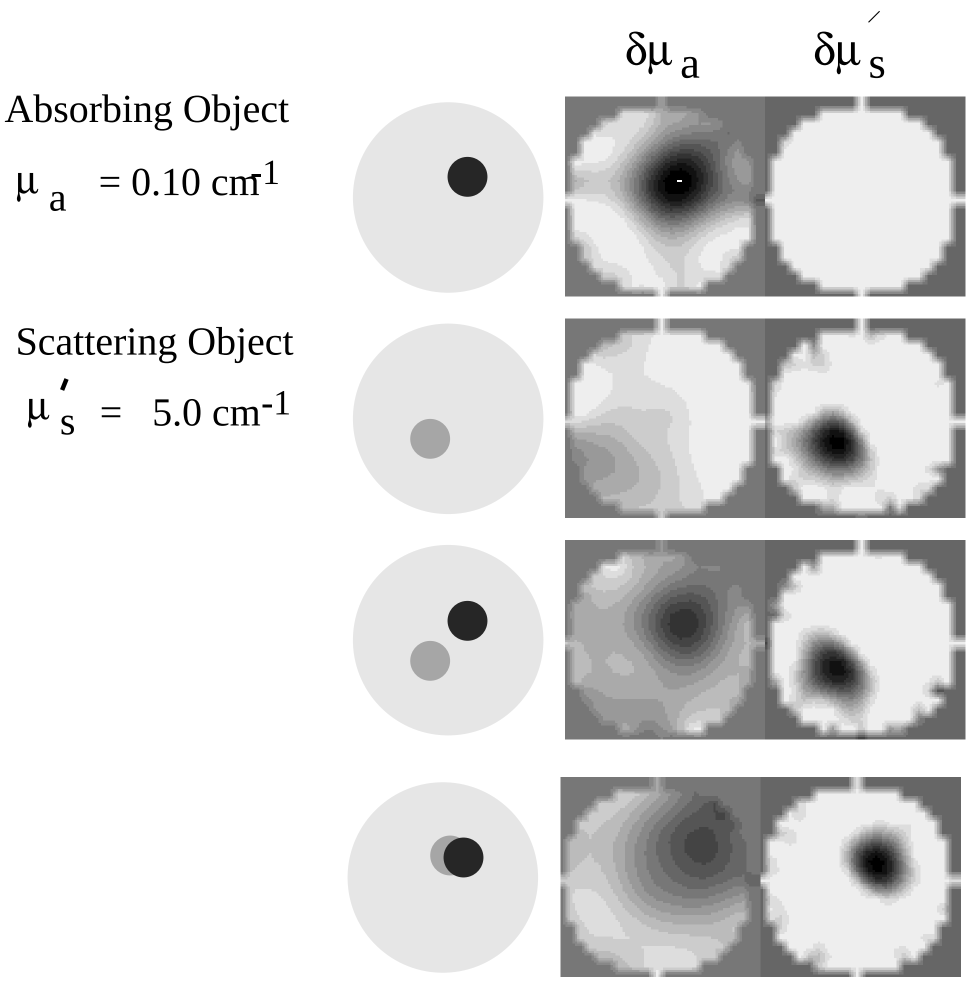

Figure 6.1: Simultaneous reconstructions of absorption and scattering.

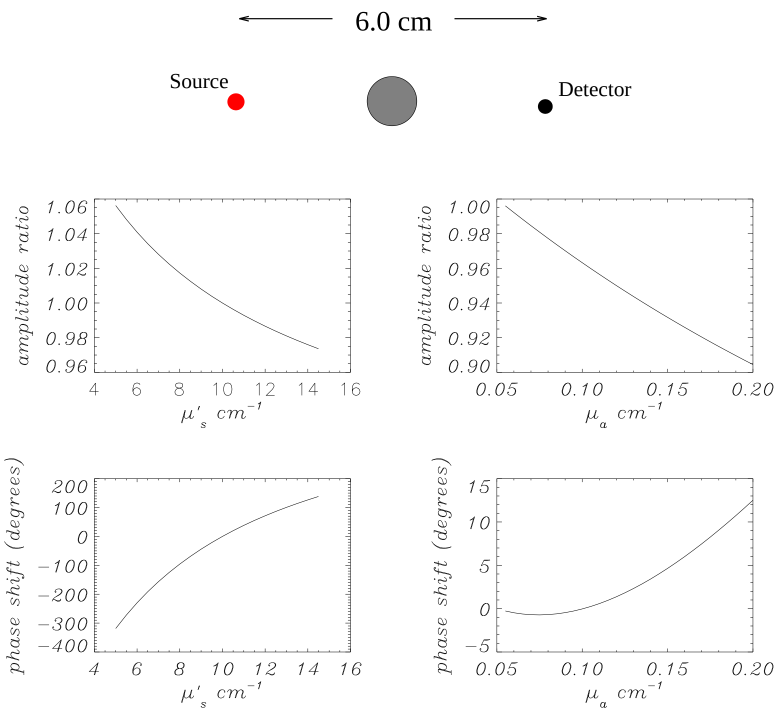

Figure 6.2: Effect of absorption versus scattering variation on amplitude and phase. At 500 MHz, the amplitude ratio (i.e. amplitude with inhomogeneity relative to amplitude without inhomogeneity) and phase change for various objects with either a pure scattering change or absorption change. Scattering changes affect the phase a great deal more than an absorption change.

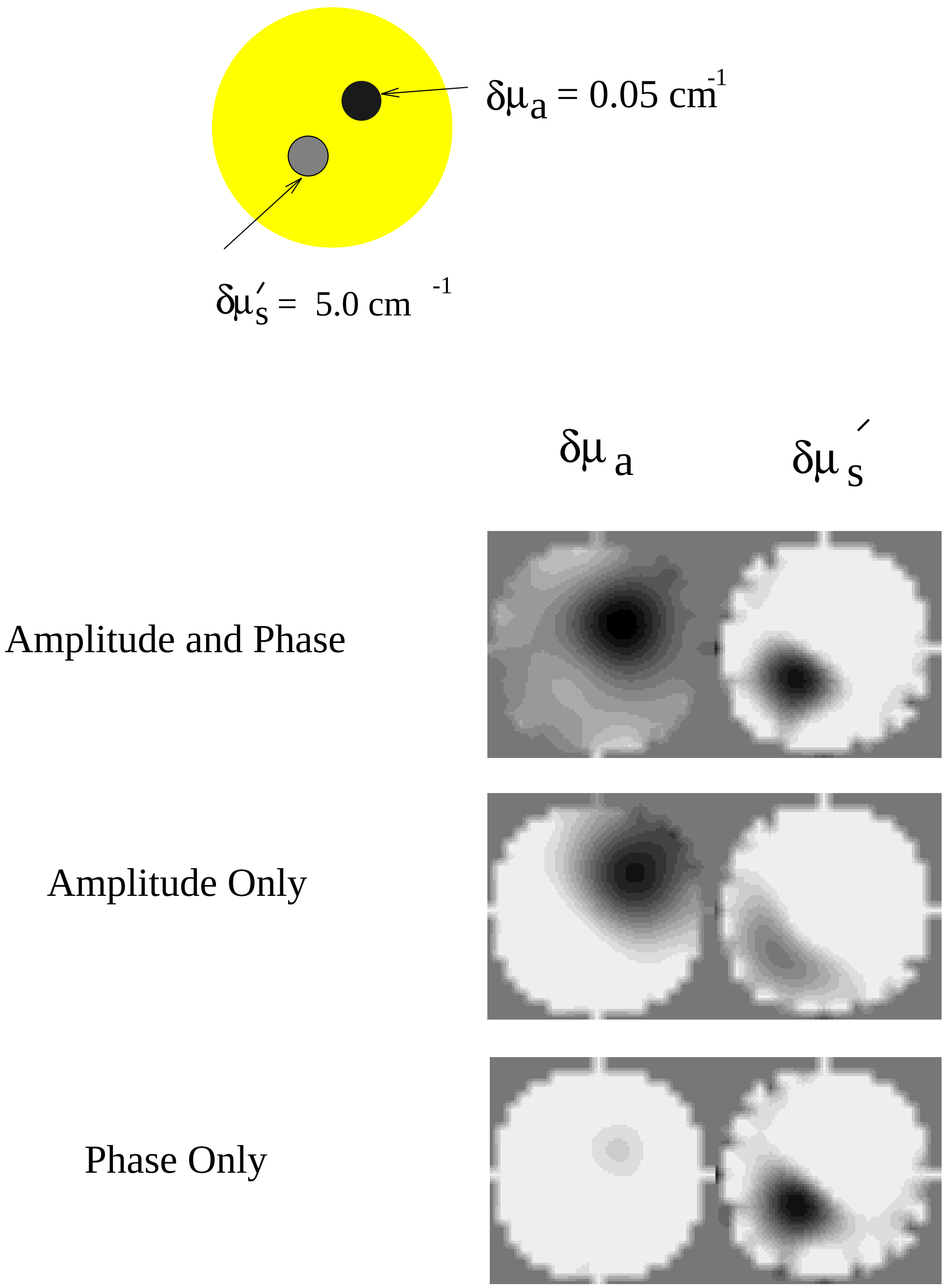

Figure 6.3: Using the Rytov expansion, the amplitude and phase can be used separately to reconstruct the absorption and scattering. When both amplitude and phase are used (top) the reconstructions give the correct positions of the object. When the amplitude data alone is used (middle), both objects are seen at slightly offset positions. However, the phase information alone (bottom) gives a very good reconstruction of the scattering for this high modulation frequency (500 MHz).