Symmetric Image Normalization in the Diffeomorphic Space

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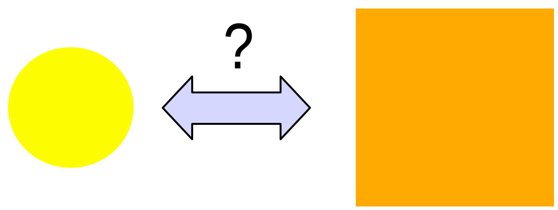
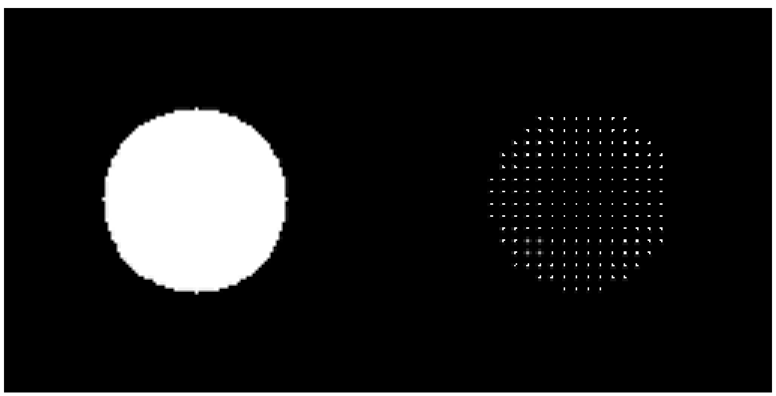
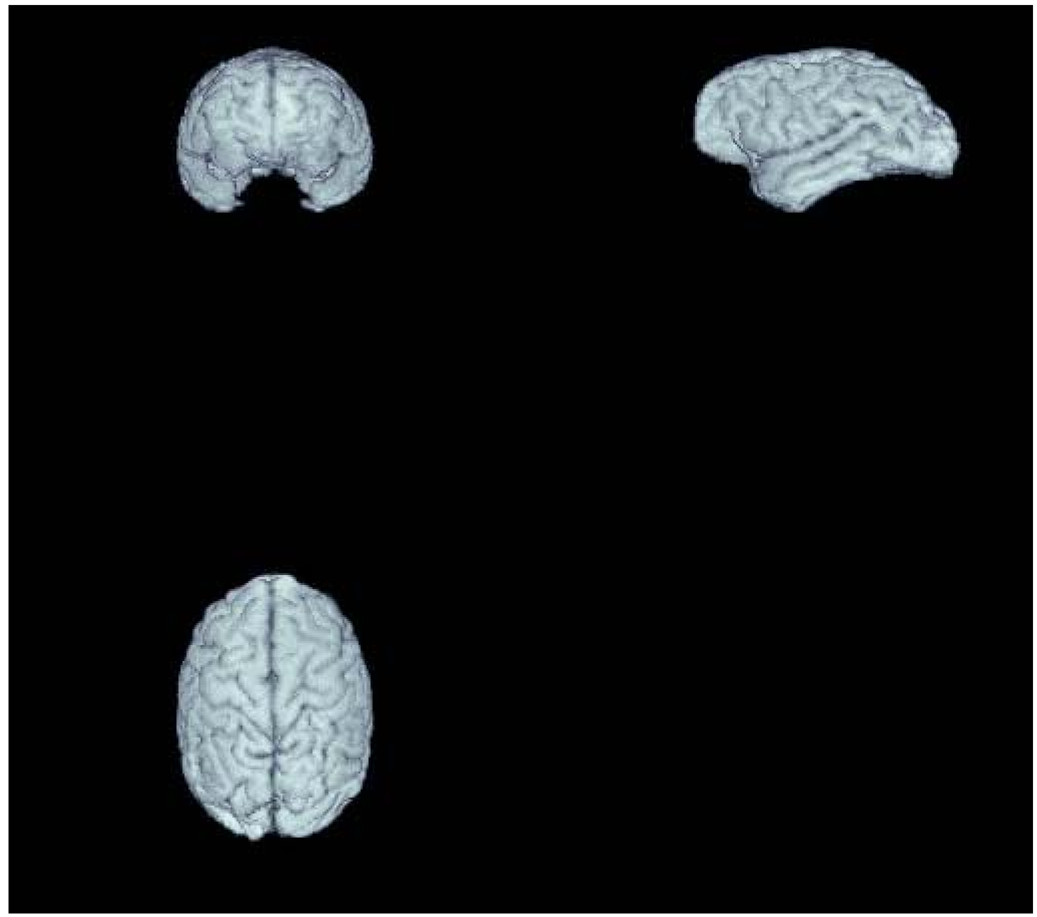
Normalizing Transformations Quantify Local Shape and Size Change





Circle to Square   Transformation

Shape Difference



Chimp to Human

Optimizing the Normalizations over Diffeomorphisms

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- Removing topology preserving variability ideally situates one to study the residual differences in non-topology preserving anatomic variability

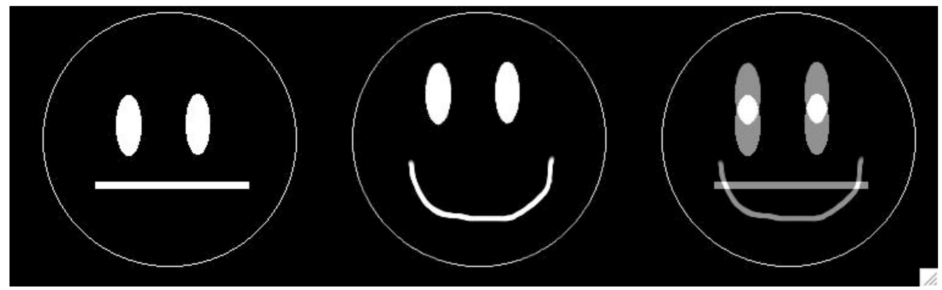
Diffeomorphic Normalization
Lagrangian Push Forward Algorithm

$$E_{LDDMM}(I,J) = \inf_{\phi_1} \int_{t=0}^{1} \|v_1\|_L^2 dt + \int_\Omega \omega |I(\phi_1(1)) - J|^2 d\Omega.$$

Subject to:

$\phi_1 \in \mathit{Diff}_0$ the solution of:

$d\phi_1/dt = v_1(\phi_1(t))$ with $\phi_1(0) = \mathbf{Id}$.



Avants et al, Med Imag Anal, 2005

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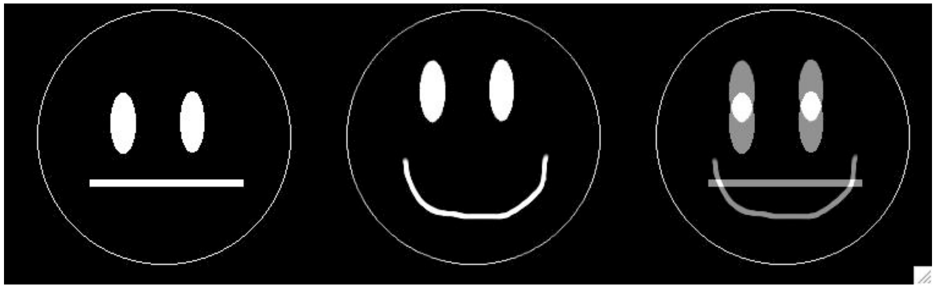

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Total deformation, summed in time, gives metric distance in manifold space of *Diff*

except dist(I,J)≠dist(J,I):

LPF is asymmetric

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  - Dependence upon registration directionality introduces bias
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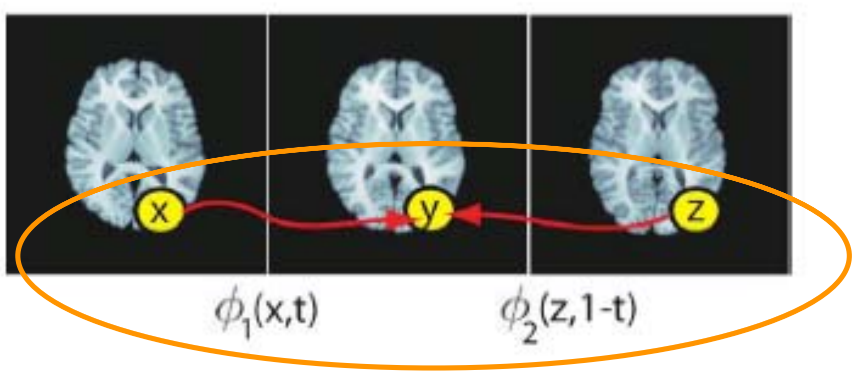
- Solution: formulate our computation such that the tangents defining the geodesic are derived symmetrically with respect to *I* and *J*

Symmetric Normalization

- Find the shortest diffeomorphism between I, J such that $\phi_1(t)I = \phi_2(1-t)J$, where the geodesic is parameterized as in Eqn 1



$$E_{sym}(I,J) = \inf_{\phi_1} \inf_{\phi_2} \int_{t=0}^{0.5} \{\|v_1\|_L^2 + \|v_2\|_L^2\} dt + \int_\Omega \omega |I(\phi_1(0.5)) - J(\phi_2(0.5))|^2 d\Omega.$$

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$$\begin{aligned} \phi_1(\mathbf{x},1)I &= J, \\ \phi_2^{-1}(\phi_1(\mathbf{x},t),1-t)I &= J, \\ \phi_2(\phi_2^{-1}(\phi_1(\mathbf{x},t),1-t),1-t)I &= \phi_2(\mathbf{z},1-t)J, \\ \phi_1(\mathbf{x},t)I &= \phi_2(\mathbf{z},1-t)J, \qquad (1) \end{aligned}$$

Coordinate System Invariant Parameterization of Geodesic

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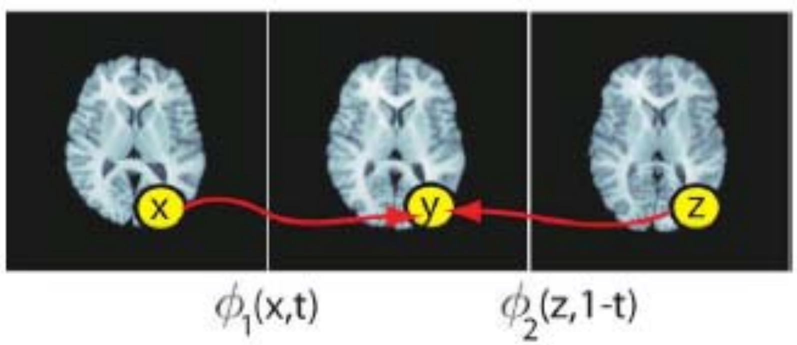

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$\phi_1(x,t)$ $\phi_2(z,1-t)$

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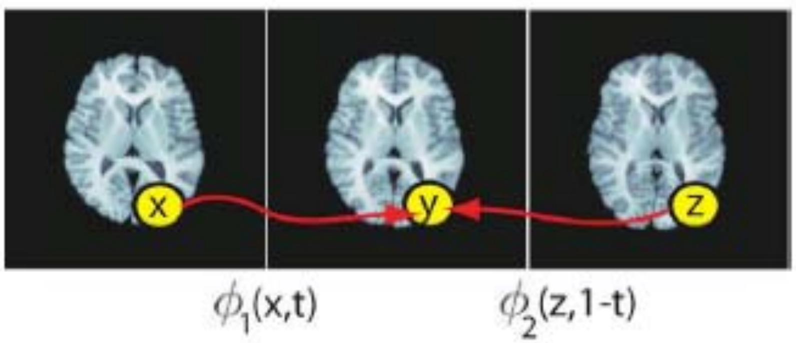

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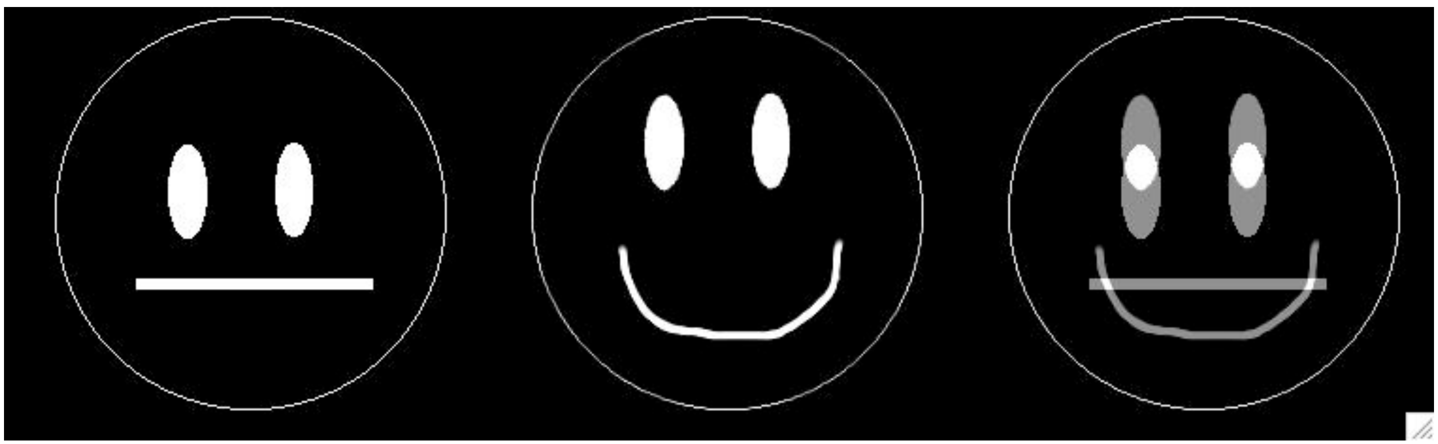


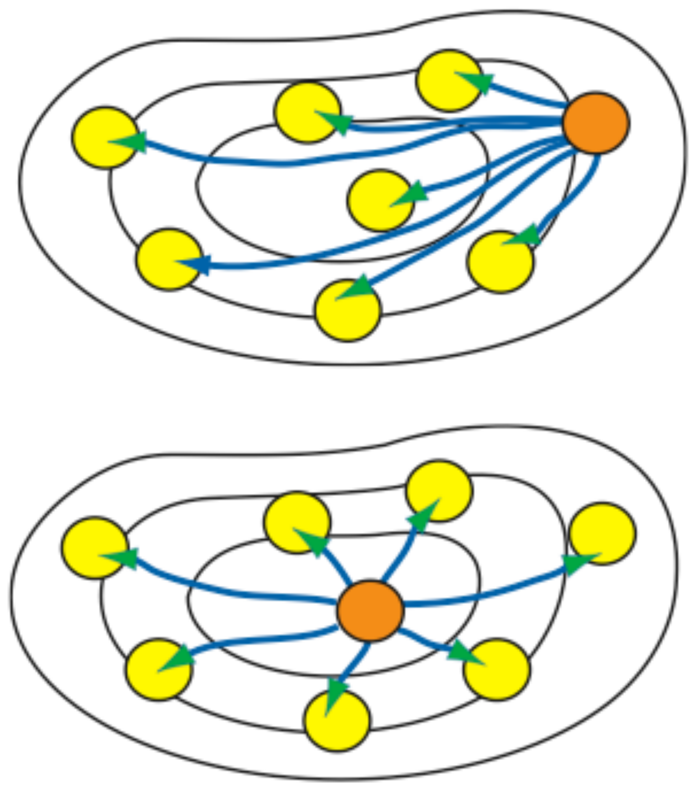
Sad Happy Overlay

Extension to Symmetric Population Studies
Optimal Template Construction

- Find the template and set of transformations that gives the “smallest” parameterization of the dataset:

$$\sum_i \inf_{\phi_1^i} \inf_{\phi_2^i} \int_{t=0}^{1} \{ \|v_1^i\|_L^2 + \|v_2^i\|_L^2 \} \, dt + \int_\Omega \omega |\bar{I}(\phi_1^i(0.5)) - J_i(\phi_2^i(0.5))|^2 d\Omega.$$

where $\forall i, \phi_1^i(0) = \psi, \bar{I}(\phi_1^i(\mathrm{x},1)) = J_i$, and each pairwise problem is solved with SyN (see equation 1 and 2).



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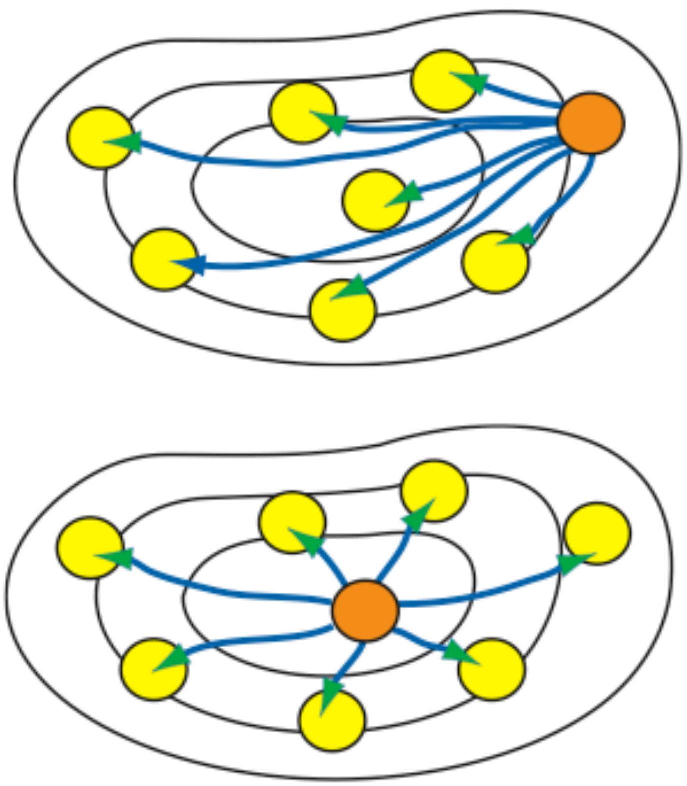

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