REPORT OF THE LOW-FIELD GROUP: THE MAGNETOCARDIOGRAM

December, 1975
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Prepared by D. Cohen
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MAGNETIC FIELD PRODUCED BY A CURRENT DIPOLE

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SUMMARY

The current dipole is the current analogue of the charge dipole; it can be used in the solution of the magnetic forward and inverse problems, similar to the way the charge dipole is used in the solution of the potential forward and inverse problems. In general, the magnetic field produced by the current dipole is due both to the current generated in the volume conductor by the dipole, and to the dipole element itself. For some special shapes of volume conductor, however, one or more of the magnetic field components receives zero net contribution from the volume conductor current, therefore is entirely due to the dipole element. This applies to all field components in an infinite volume conductor and the component of external field which is normal to the surface of these configurations: a semi-infinite volume, an infinite conducting slab, and a spherical volume conductor. This property, which allows great simplification in the solution of the magnetic forward and inverse problems, is discussed theoretically; it is also proven experimentally by the use of electrolytic tank measurements. Applications of this property are discussed in terms of the magnetic detection of dipole combinations which have a suppressed and often undetectable surface potential. Based on this property, a new method of displaying the distribution of normal field component over a surface is presented; a two-dimensional gradient of the normal component is displayed, by means of an array of arrows. This method allows some simple combinations of source dipoles to be visually determined, as simple solutions to the magnetic inverse problem. In the appendix a description is given of an earlier experiment on the canine heart which is interpreted by using the dipole concepts developed here.

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INTRODUCTION

In recent years various measurements have been made of the heart's magnetic field over the chest. The first measured distributions were displayed as a time-sequence of magnetic vectors distributed over the chest; these were from normal subjects, only during QRS. More recently, distributions of the single magnetic component which is normal to the skin have been shown for the entire heartbeat. These were recorded from subjects with both normal and abnormal hearts; they were displayed as an array of magnetocardiogram (MCG) traces over the chest. These efforts have been partially justified on the basis that the heart's magnetic field can contain information either unavailable in the surface potentials, or too difficult to extract from the potentials with surface electrodes. Now that the magnetic fields have been measured and displayed, the next step is to extract any new information. A preliminary approach to this problem, recently used, has been visually to compare the magnetic data with the electrocardiograms (ECG's). However, for more utility, a solution of the heart generators which produced the field must ultimately be found; this is the magnetic inverse solution. In beginning our program to evolve various inverse solutions we have been studying the current dipole.

The current dipole produces the same current distribution in the volume conductor as does the charge dipole, and is the basic element used in mathematically simulating biological current generators. Although we here deal only with the discrete current dipole, it has been shown that a continuous current dipole distribution can exactly simulate the membrane current generators at the cellular level. By summation of cells, therefore, a continuous current distribution can exactly simulate the current generators at the macroscopic level, say of the entire heart.

The current dipole is involved in the magnetic inverse solution for the same reason that the charge dipole is involved in the potential inverse solution, that is, in the solution to the problem of determining the heart generators which produce the potentials on the torso surface. In the potential case it is impossible to solve uniquely for the charges in the heart which are the sources of the potential; instead, solutions are obtained of assumed models of the sources, consisting of either discrete charge dipoles or charge multiple expansions. Magnetically, it is also impossible to obtain a unique inverse solution for the current generators which are the source of the field, and one must again use models to approximate these current sources. The simplest element with which to construct models of source currents is the current dipole. We have therefore been studying the magnetic field of the current dipole located in several elementary shapes of volume conductor; this is actually a simple magnetic forward problem. However, solving simple forward problems allows one to use these arrangements in the inverse...
solution. Although models using current dipoles have already recently been used in the complex forward problem of simulating the human heart within the torso, some of the fundamental magnetic properties of a single dipole were not considered, such as the property which will here be dealt with.

The current dipole is illustrated in Fig. 1. The dipole generates ion current in the surrounding volume conductor called the volume current. Generally,

\[ \mathbf{B} = \mathbf{B}_v + \mathbf{B}_d \]

Fig. 1. The current dipole of length \( d \) (heavy arrow) in an infinite volume conductor generates a total current \( i \) called the volume current. At any point the magnetic field (heavy circles) receives a net contribution of zero from all the volume current; the field is therefore due only to the dipole element \( i d \). The section of hemispherical cap (broken line) is used in the derivation of the total magnetic field of the dipole.

the magnetic field is produced both by this volume current and by the current element of the dipole itself. We express this as

The objectives of
this paper are: first, to demonstrate this unusual and important property of the current dipole both theoretically and experimentally; second, to discuss its applications to the forward and inverse problem of the heart's magnetic field. Also, we present a method of displaying the distribution of $B_z$ over a surface when this property is valid. Finally (in the appendix) we present an earlier experiment on the dog heart which we here interpret by using the dipole concept.

In the third paper in this group we change from these special shapes of volume conductor to the shape of the human torso; it is shown that this special property is still approximately maintained. In the fourth paper we use this property to solve, in a simple way, the inverse magnetic problem from the MCG maps of several subjects.

THEORY OF THE CURRENT DIPOLE

The current dipole is a current generator in the form of a directed line element $\Delta \vec{z}$, which generates a total current $i$ in the surrounding volume conductor; this is shown in Fig. 1. By definition $\Delta \vec{z} \cdot \vec{r}$ where $\Delta \vec{z}$ is the dipole length and $\vec{r}$ is the distance from the dipole to any measuring point $P$. The quantity $i \Delta \vec{z}$ is finite and is called the dipole moment; the magnitude $i \Delta \vec{z}$ is here called the dipole element. The current dipole is distinct from a magnetic dipole, which is a small current loop. The current dipole is the current analogue of the charge dipole, which consists of charges $+q$ and $-q$ and is described by the dipole moment $q \Delta \vec{z}$. The two dipoles are physically identical but are described differently. In an infinite homogeneous volume conductor, the current dipole generates a current density

$$\mathbf{J} = -\hat{r} \left( \frac{i \Delta \vec{z} \cdot \hat{r}}{4\pi r^2} \right) = \frac{i \Delta \vec{z}}{4\pi r^3} \left( 2 \cos \theta - 2 \sin \theta \right)$$  \hspace{1cm} (2)

where $\hat{r}$ is the radial unit vector, $\hat{\theta}$ is the unit vector in latitudinal direction $\theta$; MKS units are used in this theory section. For the charge dipole, $\mathbf{J}$ in (2) is replaced by $\sigma \mathbf{E}$ and $i$ by $q/k$, where $\sigma$ is the medium conductivity, $\mathbf{E}$ is the electric field, and $k$ is the permittivity of the medium.

We here wish to describe the magnetic field of the dipole in various shapes of volume conductor; we begin with the simplest case, that of the homogeneous infinite volume conductor. There are several different methods we can use to derive this field. Baule, in his thesis, briefly considered the current dipole and used a standard method, which we also use here. To the section of spherical cap shown in Fig. 1 we apply the relationship (sometimes called Ampere's circuit law):

$$\oint \hat{B} \cdot \hat{p} = \mu_0 \int \mathbf{J} \cdot d\vec{s}$$  \hspace{1cm} (3)

where $\hat{B}$ is the field at the perimeter of the cap, $\hat{p}$ is the distance along the
perimeter, $\delta s$ is an element of cap surface area, and $\mu_0$ is the permeability of free space; the medium permeability is everywhere assumed to be unity. By putting $\bar{J}$ from (2) into (3), we get:

$$B_\phi(2\pi r\sin \theta_0) = \mu_0 \int_0^{2\pi} \int_0^{\theta_0} \frac{i\delta \alpha(2\cos \theta)}{4\pi r^3} r^2 \sin \theta d\theta d\phi$$  \hspace{1cm} (4)

where $\theta_0$ is the angle to the perimeter and $\phi$ is the longitudinal angle. This yields

$$B_\phi = \frac{\mu_0 i\delta \alpha \sin \theta_0}{4\pi r^2}$$  \hspace{1cm} (5)

where we will let $B_\phi$ be simply called $B$. The other two components are zero due to current symmetry. This is mathematically identical to the differential form of the Biot-Savart law which gives the element of field produced by an element of current. By comparison, therefore, the field at any point in the volume conductor is due only to the dipole element, or $\bar{B} = \bar{B}_d$; the net field from the volume current is zero, or $\bar{B}_v = 0$. This unusual property is important, since it can be extended to some conductor configurations of more applicability. Unfortunately, Baule did not grasp or comment on the significance of this result and it was not published, in contrast to other portions of the thesis.

After we had verified this property experimentally, discussions led to some new theoretical proofs of it. The following proof is quite simple and is readily extended to other shapes of volume conductor; the basic idea was suggested by Prof. D. W. Kerst of the University of Wisconsin. The volume current of the dipole in an infinite homogeneous volume conductor can be considered to be the linear superposition of current from each pole separately; these currents, illustrated in Fig. 2, are connected with the electric field from each pole separately and

Fig. 2. An instructive way of considering the current dipole in an infinite volume conductor. The volume current, for example as in Fig. 1, is a linear superposition of straight-line current due to source and sink separately; these are shown as light, radiating lines. By a symmetry argument, the field of each separately is zero, hence zero when combined. A side boundary is added later, shown as the broken line.
are therefore radiating straight lines. We now consider the magnetic field at point \( P \) and define three orthogonal components of field at that point: \( B_x \) is perpendicular to this plane, out of the page. We then note that all three magnetic components must be zero because of current symmetry; each current element from the source which contributes to any magnetic component has a counterpart which produces an equal and opposite contribution to this component. It follows that the total magnetic field, which is the sum from both source and sink, is zero. The magnetic field therefore is only due to the dipole element \( \Delta l \). A field from a current element is given by the Biot-Savart law, which is identical to (5), as we wished to show.

The above argument does not depend on the condition \( \Delta l \ll r \). It depends only on the existence of both a source and sink, hence it also applies to a dipole (poles far apart); it therefore is more general than the previous derivation. However, in both methods only dc can be used, otherwise \( \mathbf{E} \) would be a function of the time-varying electric field as well as the current. It is important to note that the source and sink must always be considered as a member of a pair; Maxwell’s equations are derived from experiment and can give false results when applied to a single current pole, not experimentally possible. In the above argument we have avoided Maxwell’s equations by using only a symmetry argument for each pole.

We can now extend this method to the case of a semi-infinite volume conductor. Let the boundary coincide with the page, and let the dipole be below and parallel to the page. We now move the point \( P \) from its previous location to a new location above the paper, in the air, by a translation upward; \( B_0 \) and \( B_z \) are therefore unchanged, but \( B_x \) now changes from the radial to the projected radial direction, parallel to the page; it will be called \( B_x' \) now. From the top view of Fig. 2 these axes appear to be unchanged. We note again that \( B_x' = B_z = 0 \) because of current symmetry of source current, but \( B_y \neq 0 \). In considering the contribution of the sink current to the field at \( P \), we see again \( B_z = 0 \) but \( B_x \neq 0 \) because its direction is only radial to the source, not to the sink; therefore, for the sum of source and sink, \( B_{xv} = 0 \) but \( B_x \neq 0 \) and \( B_y \neq 0 \). This argument holds as well for two parallel boundaries, that is, for an infinite slab. For the case, therefore, of a dipole in a semi-infinite volume conductor, or in an infinite conducting slab, \( B_x = B_{2d} \).

This method can also be used for several shapes of volume conductor which are more complex. Consider, for example, a slab to which is added a perpendicular side boundary as shown by the broken line in Fig. 2. Using the same argument as in the previous case, \( B_{xv} = 0 \) directly above the centerline of the dipole; in this case only a dipole can be used, not a dipole. This result is unchanged if a second side boundary is added on the other side of the dipole, parallel to the other boundary, producing an infinitely long rectangular bar of volume conductor. A dipole or dipole within these shapes begins to approach the configuration of a cardiac generator within the torso.

A second method which is useful for the semi-infinite volume conductor is the method of images. The current distribution produced by a current dipole
parallel to and below the surface, can be exactly simulated in the following way:
this current dipole is first envisioned in an infinite volume conductor, with only
a hypothetical surface at the location of the boundary; another dipole (called the
image dipole) is now added at a location above the hypothetical surface and equi-
distant to it, and oriented similar to the original dipole. The current distribu-
tion below the hypothetical surface must now be that of the original dipole in
the semi-infinite conductor; this is because there is now no normal component of
current at the hypothetical surface. The tangential component, however, is dou-
bled. This is illustrated in Fig. 3 at point 1 on the right. At this point \( \vec{J}_D \)
is the current density due to the actual lower dipole in the infinite conductor,
\( \vec{J}_I \) is the current density in the infinite conductor due to the image, and the
total \( \vec{J}_T \) is seen to be tangential to the surface with twice the length of either
dipole separately. We now completely remove the conductor above the surface but
leave the current density below the surface unaltered, thereby simulating exactly
the original arrangement of the dipole in a semi-infinite volume conductor. The
removal of this upper current does not change the current density at point 1,
which remain as shown.

![Fig. 3. Current dipole in a semi-infinite volume conductor. The image dipole (broken) is shown equidistant above the surface. At point 1 the current densities \( \vec{J} \) are shown due to the dipole, the image, and the total of the two. Points 2 and 3 are used in the Gesselowitz integral (8).](image)

Before considering the magnetic field involving the image, we digress for a
moment to point out a useful property of the surface potential. Because \( \vec{J}_T \) in
the semi-infinite case is twice the density of the infinite volume conductor at
that same point, it follows that the surface potential is everywhere doubled due
to the boundary. Stated otherwise, the insertion of a boundary to an infinite
volume conductor doubles the potential everywhere over the area of the surface;
this is independent of the conductivity of the conducting medium. This fact is
considered in the next section of this report, in which heart potentials are dealt
with on the torso surface, and a comparison is made between the effect of the sur-
face on the potential and on the normal component of magnetic field.

Returning to the magnetic field, we can now proceed with the image argument
in order to deduce the normal component of field at the surface. We consider
point 2 on the upper left of Fig. 3, and move it downward so that it coincides
with the surface. With both the dipole and its image present in the infinite

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conductor (before we removed the upper portion) $B_z$ has been doubled in comparison to the single original dipole in the infinite conductor. This is because the currents must be symmetrical on both sides of the hypothetical plane. We now completely remove the upper current; $B_z$ must therefore drop to half the value, hence to the original value due to the lower dipole in the infinite conductor. It follows that $B_z$ "does not know" about the surface, and simply obeys equation (5); stated otherwise $B_z = B_{zd}$. We note that this proof is not as general as the previous (dipole) proof, since this only holds at the surface while the previous proof applies to $B_z$ at any distance above the surface. This image treatment was also given in Baule's thesis\textsuperscript{10}, although again he did not point out the significance of this result.

As pointed out to us by Dr. Neil Cuffin, a third method which is useful for deriving $B_z$ for the semi-infinite volume conductor, is due to Geselowitz\textsuperscript{11}. In this method, the external magnetic field produced only by the volume current in a homogeneous conductor is given by the integral

$$\mathbf{B}_v = \frac{\mu_0}{4\pi} \int_S \frac{\mathbf{V} \times \mathbf{r}}{r^2} \times d\mathbf{s} \quad (6)$$

where $V$ is the potential at the surface source point, $\mathbf{r}$ is the vector from this point to the external field point, and $d\mathbf{s}$ is an element of surface. This is illustrated on the left side of Fig. 3; point 3 is the surface source point and point 2 is the field point. It is seen that the cross-product can only be parallel to the surface, hence $B_{zv} = 0$ from which it follows that $B_z = B_{zd}$. This method can be extended to the case of a spherical volume conductor; this is yet another case where $B_{zv} = 0$ ($z$ is here the normal, radial direction to the surface). This can again be seen by the integral (6). Because $d\mathbf{s}$ is radial the cross-product can only be tangential to the spherical surface; hence at any distance from the sphere, for the component normal to the surface, $B_{zv} = 0$ therefore $B_z = B_{zd}$.

Yet another method is occasionally useful. As pointed out to us by Geselowitz, the lead-field method of McFee (as given by Plonsey\textsuperscript{4}) can be used for the semi-infinite volume conductor. Briefly, the current induced in the conductor by an external magnetic detector, say a single loop, is everywhere parallel to the surface; by reciprocity, a current dipole oriented along the line of induced current will induce a voltage in the detector without surface interaction, hence the volume current cannot be involved; the volume current "knows" about the surface.

The property of $B_{zv} = 0$ can be visualized in the following way. Consider first an imaginary plane in an infinite volume conductor, and a dipole parallel to this plane, and below it. Consider also the magnetic field above the plane due to this dipole. We then replace the imaginary plane by an actual boundary, with air above the boundary. Due to this boundary a new, special magnetic field now appears. However, this field is always parallel to the boundary, at any distance above the boundary; its particular distribution is unimportant here. The original normal component $B_z = B_{zd}$ is unchanged. If yet another boundary would be added, parallel to the first but below the dipole, another new field would be added in
the air above; but again this field would only be parallel to the boundaries. The same visualization can be applied to the addition of a spherical boundary to an infinite volume conductor; the new field due to the spherical boundary is always tangential to the boundary; there is no normal or radial component.

The significance of \( B_{yz} = 0 \) for the above configurations is that only the dipole elements and (5) are involved in the solution of both the forward and inverse magnetic problem, if only the \( x \)-component of field is used. The use of (5) for \( B_z \) is now similar to the use of the equation for the surface potential due to a charge dipole; some special boundaries, as in the semi-infinite case, only affect the magnitude but not the distribution of surface potential, which is then determined only by the charge dipole. However there is one major difference between \( B_z \) from a dipole and \( V \) from the same dipole: the two distributions differ by a 90° angular shift from the dipole direction; they are complementary, not identical. The application of this difference is the following. There are some arrangements of dipoles which yield greatly diminished surface potentials, compared with one dipole; for these arrangements \( B_z \) may not be diminished, and may even be enhanced. This is due to the 90° rotation. Because of noise, these arrangements may be hidden from potential measurements, but readily revealed by \( B_z \) measurements. A basic arrangement of this type is shown in Fig. 8A and is discussed further on.

Because of the potential usefulness and the unusual nature of \( B_{yz} = 0 \), we considered it worthwhile to verify this property experimentally for the case of the semi-infinite volume conductor. The measurements are described further on, but we here explain one aspect of \( B_z \) which is necessary to the understanding of these measurements; this concerns those quantities related to \( B_z \) which are useful to measure. We examine the curves of Fig. 4; these are the values of \( B_z \) and of its derivatives (as well as \( B_y \)) in a plane, above a single dipole, calculated from (5). We note that \( B_z = 0 \) just over the dipole, and has its maximum value at the sides. However, the quantities \( \partial B_z / \partial y \) and \( B_y \) are maximal immediately over the dipole. Because of this, for some experiments \( \partial B_z / \partial y \) is the best quantity.

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**Fig. 4.** Quantities computed from equation (5), using the arrangement shown in the upper left. The quantity \( \partial B_z / \partial y \) is a maximum immediately over the dipole and is therefore the most useful quantity to measure in some experiments. The quantity \( B_y \) is shown only to demonstrate, by comparison, the higher resolution of \( \partial B_z / \partial y \). These quantities are shown again in Fig. 11, on a different scale and for a different purpose.
to measure. $\frac{\partial B_y}{\partial y}$ is shown only as a reference, to demonstrate the higher resolution of $\frac{\partial B_z}{\partial y}$. For example, in determining the decrease of $B_z$ with $z$-distance from the dipole as one check of (5), the direct measure of $B_z$ would necessitate a complicated $y$-placement of the detector. However, the measurement of falloff of $\frac{\partial B_z}{\partial y}$ with increasing $z$ need simply be made on the vertical line $x=0$. For this measurement, by differentiation of (5) it can be seen that $\frac{\partial B_z}{\partial y}$ should vary as $z^{-3}$; this is a suitable falloff for experimental verification. In the experiments described below, $B_z$ was measured in some of the experimental arrangements, while $\frac{\partial B_z}{\partial y}$ was measured in others.

MATERIALS AND METHODS

We here describe the experiments which were performed (by H.H.) in order to verify that $B_{zv}=0$ from a dipole in a semi-infinite medium. Verification of this property for this case was considered to be verification of the principles which apply to all shapes which possess this property. The semi-infinite arrangement was simulated by electrodes in an electrolytic tank. The basic idea was to have a choice of two different generators: a current dipole complete with volume current, or only the volume current where the dipole element $dI$ was absent. These are shown in Fig. 5 as electrodes fed by wire pair a or b, respectively.

Fig. 5. Electrolytic tank arrangement used to verify $B_{zv}=0$. The two electrodes, shown with exaggerated separation, were fed ac by either pair of insulated wires, a or b. The quantities $\frac{\partial B_z}{\partial y}$ were measured above the surface by the voltage induced in either one coil or both coils in series-opposition. A complete current dipole was simulated when pair a was used; pair b gave only the return current because the vertical wires produced zero $B_z$. The shaded rectangle is an insulated sheet used later to simulate a flat boundary.
When using the a-pair, both the dipole element and return current were present; the dipole element consisted of the current in the section of wire between the two electrodes. When using only the b-pair, the vertical wires gave zero contribution to $B_z$ and the horizontal section at the bottom produced negligible field at the surface; the only possible contribution to $B_z$ was the volume current in the electrolyte. The choice of a-pair or b-pair allowed a range of convincing experiments to be performed.

The tank used for most of the measurements was plastic, roughly cubical, and about 1.5 meters on a side. Smaller tanks were used for some of the measurements. The conductor was a simple NaCl solution with resistivity of about 30 ohm-cm. The electrodes were solder spheres separated by a distance which could be varied from 0.5 to 1.5 cm, and were fed by formvar-coated, stiff magnet wire. The current was ac of 3-kHz frequency, generated at a level of about 20 ma(rms) by an audio amplifier fed by an oscillator. This system was completely linear and gave a large detected magnetic signal relative to noise. Two identical coils were used as detectors, by means of the voltage induced by the alternating $B_z$. Each coil contained about 500 turns of thin magnet wire and were 2.5 cm in diameter and 1.6 cm in length. To measure $B_z$ only one coil was used; both coils in series constituted a gradiometer which measured the gradient $\partial B_z/\partial y$. The gradient was measured both for the reason explained in the theory section, and because any background signal was eliminated without effort due to cancellation. One or both coils fed a PAR tuned amplifier operating at $Q = 100$. The rectified output signal was proportional either to $B_z$ or to $\partial B_z/\partial y$. A number of different experiments were performed, and five of these are reported here.

The first experiment consisted of a comparison between $\partial B_z/\partial y$ from the a-pair and from an equivalent dipole element completely in air, without volume current. The air arrangement and experimental results are shown in Fig. 6. The dipole element consists of the short, horizontal section of wire. The purpose of this experiment was twofold: first, to calibrate and assess any problems in both the current system and detection system, using only a known 14% in air, without volume current. Second, as one method of detecting any significant contribution from the volume current in the tank; if $B_{zv} = 0$, then the air and tank measurement would yield identical values of $\partial B_z/\partial y$, measured as a function of $y$. Care was taken with the wire shape in air to insure that the vertical members were straight, hence gave no contribution to $B_z$; to accomplish this, the vertical wires were run through long ceramic pipes of very small bore which then encased the wires and allowed fine adjustment of position and angle.

In the second experiment a comparison was made between $\partial B_z/\partial y$ due to the b-pair and to the a-pair, hence between $B_{zv}$ and $B_{zd}$. (A similar comparison using $B_z$ is part of the fifth experiment). The inter-electrode spacing was identical in each pair, and care was taken to insure that the vertical b-wires were straight. The measurements of $\partial B_z/\partial y$ were made just above the surface at three locations: at $x = y = 0$, at $x = 0$, $y = 40$ cm, and at $y = 0$, $x = 40$ cm. This is a sensitive test of $B_{zv} = \ell$; if true, the ratio of $\partial B_z/\partial y$ from
b-pair/a-pair would be <<1. The data from these measurements are stated in the result section, further on.

Fig. 6. Measurement of $\frac{\partial B_z}{\partial y}$ due to a horizontal current element in air, and due to the a-pair in the tank. The same current $i$ and length $\Delta l$ were used in both. In air, each of the two coils of the gradiometer received a contribution only from the horizontal element; the two vertical wires each produced $B_z = 0$ everywhere.

The third measurement was the falloff of $\frac{\partial B_z}{\partial y}$ with increasing distance $z$, on the line $x=y=0$. Only the a-pair was used here. If $B_{yz} = 0$ then (5) is valid and yields, by differentiation, the $z^{-3}$ falloff. A measured curve and a calculated curve are shown in Fig. 7.

The fourth experiment was performed in order to find the effect on $B_z$ of a flat side boundary. The boundary was simulated by an insulating sheet of plastic, about 4 cm by 10 cm in area; it was first arranged in the $x$-$y$ plane as illustrated in Fig. 5, then later in the $x$-$z$ plane. The information obtained relates to the human heart and the boundaries of the high-resistivity lung; these boundaries will produce a volume-current contribution to the magnetic component normal to the chest. Although the actual lung boundaries are curved and complicated, information from a flat boundary will be useful in eventually understanding the more complicated shapes.
Fig. 9. Arrangement for measuring the effect of a cylindrical (insulating) boundary, in the electrolytic tank. The cylinder extends up to the liquid surface. The quantity \( B_z \) is measured with the single coil; measurements are made at \( z = 0 \), along the y-axis. Results are shown in Fig. 10.

The top was just above the liquid surface; the electrodes were centered on the axis. For accurate alignment and sturdy support, a somewhat smaller tank was used than in the previous experiments. The quantity \( B_z \) was directly measured here, at a fixed \( x \) and \( z \), but as a function of \( y \). As before, the b-wires were used to obtain a pure \( B_{2y} \); the a-wires were used to yield pure \( B_{2d} \) when the cylinder was removed, or total \( B_z \) with the cylinder in place. In this experiment the phase of the detected signal with respect to phase of the oscillator was noted; this gave the relative polarity of \( B_z \). Curves of \( B_z \) vs. \( y \) were obtained for these four arrangements: a-wires without cylinder, with the intent of yielding \( B_{2d} \); b-wires without cylinder, to yield \( B_{2y} = B_{2d} = 0 \); a-wires with cylinder, to yield \( B_{2d} + B_{2y} \); b-wires with cylinder, to yield \( B_{2y} \) due to the cylinder. The measurement using b-wires without cylinder is also an extension of the second experiment, in which the gradient was measured due to b-wires only. The four measured curves are shown in Fig. 10, for two different spacings of the electrodes below the surface.

**EXPERIMENTAL RESULTS**

The gradiometer curves of Fig. 6, the results of the first experiment, are seen to be identical in air and in conducting solution, to within experimental
Fig. 8. Ratio of $B_z$ produced only by the volume current to $B_z$ produced only by the dipole element, as a function of side-boundary (insulator) spacing. The two different arrangements are shown below each graph on the same scale as the abscissa, which begins at 1.0 cm. The electrodes were 0.5 cm below the surface, and the vertical separation between the coil centers and the electrodes was 2.6 cm (reduced slightly in drawing).

spacing and orientation. Subsequently, an attempt was made to perform these experiments with a conducting plate instead of an insulator in order to simulate the higher conductivity of the blood mass in the heart; however, polarization effects at the conductor-electrolyte interface introduced large errors and these measurements were not pursued.

The purpose of the fifth experiment was to find the effect on $B_z$ of a right cylindrical boundary, oriented vertically. Again the intention was to gather information which can be used in the understanding of $B_{zv}$ due to the lung. The cylindrical boundary is midway in approximation between the previous flat boundary and the actual lung. The cylinder was an open vertical plastic tube with a radius of 4 cm and length of about 6 cm; the arrangement is shown in Fig. 9.
Fig. 7. Measured falloff of $\partial B_z / \partial y$ as a function of increasing $z$, compared with a calculated pure $z^{-\frac{3}{2}}$ curve. The falloff was along the line $x = y = 0$ from the a-pair current generator in the tank. The curves were normalized at the upper point.

The contribution to $B_{zd}$ from the flat boundary was determined relative to $B_{zd}$ in the following way: first, $B_{zd}$ was measured with one coil at $x = 0$, $y = 2.0$ cm, using the a-pair; this was the location of maximum $B_z$ for the $z$-distance used here, as can be inferred from subsequent Fig. 12B. Then, for the same measuring point, the a-pair was changed to the b-pair, always maintaining the same spacings. A check was made before the insertion of the insulator to insure that $B_z$ from the b-pair was relatively small; the experimental arrangement was considered satisfactory when $B_{zy}/B_{zd} \leq 0.02$. Then $B_z$ was measured as a function of the x-spacing between the insulator and the electrodes, as illustrated in Fig. 8A. This sequence was repeated with the detector placed at $x = 0$, $y = 1.25$ cm, which was inward from the maximum position. The entire sequence was then repeated with the insulator moved by $90^\circ$, so that it was now in the $x-z$ plane, and the $y$-position was varied; this is shown in Fig. 8B. The experimental error in the ratio $B_{zy}/B_{zd}$ (the ordinate of Fig. 8) was about $\pm 0.02$ due to small errors in

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Fig. 8. Ratio of \( B_z \) produced only by the volume current to \( B_z \) produced only by the dipole element, as a function of side-boundary (insulator) spacing. The two different arrangements are shown below each graph to the same scale as the abscissa, which begins at 1.0 cm. The electrodes were 0.5 cm below the surface, and the vertical separation between the coil centers and the electrodes was 2.6 cm (reduced slightly in drawing).

spacing and orientation. Subsequently, an attempt was made to perform these experiments with a conducting plate instead of an insulator in order to simulate the higher conductivity of the blood mass in the heart; however, polarization effects at the conductor-electrolyte interface introduced large errors and these measurements were not pursued.

The purpose of the fifth experiment was to find the effect on \( B_z \) of a right cylindrical boundary, oriented vertically. Again the intention was to gather information which can be used in the understanding of \( B_{zv} \) due to the lung. The cylindrical boundary is midway in approximation between the previous flat boundary and the actual lung. The cylinder was an open vertical plastic tube with a radius of 4 cm and length of about 6 cm; the arrangement is shown in Fig. 9.
The top was just above the liquid surface; the electrodes were centered on the axis. For accurate alignment and sturdy support, a somewhat smaller tank was used than in the previous experiments. The quantity $B_z$ was directly measured here, at a fixed $x$ and $z$, but as a function of $y$. As before, the $b$-wires were used to obtain a pure $B_{zy}$; the $a$-wires were used to yield pure $B_{zd}$ when the cylinder was removed, or total $B_z$ with the cylinder in place. In this experiment the phase of the detected signal with respect to phase of the oscillator was noted; this gave the relative polarity of $B_z$. Curves of $B_z$ vs. $y$ were obtained for these four arrangements: $a$-wires without cylinder, with the intent of yielding $B_{zd}$; $b$-wires without cylinder, to yield $B_{zy} = B_{zd} = 0$; $a$-wires with cylinder, to yield $B_{zd} + B_{zy}$; $b$-wires with cylinder, to yield $B_{zy}$ due to the cylinder. The measurement using $b$-wires without cylinder is also an extension of the second experiment, in which the gradient was measured due to $b$-wires only. The four measured curves are shown in Fig. 10, for two different spacings of the electrodes below the surface.

EXPERIMENTAL RESULTS

The gradiometer curves of Fig. 6, the results of the first experiment, are seen to be identical in air and in conducting solution, to within experimental
Fig. 10. Measurements of $B_z$ at $z=0$, with and without insulating cylindrical boundary, as shown in Fig. 9. Electrodes were centered on the cylinder axis. Both $a$-wires and $b$-wires were used. The same electrode orientations were used as in Figs. 5 and 8. In A the electrodes were 1.5 cm below the surface; this distance was increased to 3.0 cm in B. Errors were present for $y<4$ cm due to the boundary of the smaller tank used here.

error. This is our first experimental indication that $B_{zv}=0$. The curve shape is similar to the calculated $\partial B_z/\partial y$ curve in Fig. 4, due to $iA\ell$ only. The difference between the 8-cm and 10-cm spacing agrees with the $z^{-3}$ falloff.

In the second experiment, the ratio in $\partial B_z/\partial y$ of $b$-pair/$a$-pair, measured at three locations, was found to be $<0.04$, hence again $B_{zv}=0$ within experimental error. This test, extended in the fifth experiment, was direct and sensitive. In other variations of this test, we have never found any deviation from $B_{zv}=0$ or any unexplained phenomenon involved in the measurements.

Figure 7, from the third experiment, shows that $\partial B_z/\partial y$ varies as $z^{-3}$, to within experimental error. This is another confirmation that (5) is valid here. If $B_{zv}\neq 0$ then it would be unlikely that the volume current separately would have produced an inverse cubic falloff, which could also explain this experimental result. Any other rate of falloff of $B_{zv}$ would distort the measured curve from the $z^{-3}$ shape, hence this other $B_{zv}$ could be identified. A falloff in $\partial B_z/\partial y$ of $z^{-3}$ is due to a falloff in $B_z$ of $z^{-2}$ in accordance with (5); hence this
latter falloff for $B_z$ applies to a dipole in other shapes of volume conductor for which $B_{zv} = 0$ is valid. This applies, for example, to a dipole in a spherical homogeneous volume conductor, but only for the radial or normal external field component.

The results of the fourth experiment are the curves of Fig. 8; these show a strong break of radial current symmetry due to a side boundary in the semi-infinite volume conductor. In this case $B_{zv} \neq 0$, and the ratio $B_{zv}/B_{zd}$ is as large as 0.4 when the insulator is near the electrodes. For the arrangement of Fig. 8A this large ratio is understandable because the circular current symmetry of the near pole (as seen in Fig. 2) is certainly destroyed by the insulator. However in Fig. 8B the cause of the large ratio is not obvious since this a case, argued earlier, where $B_{zv}$ should be zero on the center-line. The explanation for this large value, we believe, involves the dimensions chosen in Fig. 8B, which probably violate the conditions necessary to that argument. The coil was here in a sensitive position with respect to the insulator for detecting different asymmetries in source and sink; in a sense, the coil could "see" that the asymmetry in the source was offset from the asymmetry in the sink. Other data recorded with the coil moved to the left of the electrodes confirm this; the ratio in that case is indeed much smaller than in Fig. 8B. These experiments with the flat boundary indicate that the high resistivity of the lung could produce a large value of $B_{zv}$. Also, they show that the conditions for $B_{zv} = 0$ must be stringent.

Perhaps the most direct study of $B_z$ are the curves of Figs. 10A and 10B. The uppermost curves have the same general shape as the $B_z$ curve of Fig. 4, or the solid curve of Fig. 11A, calculated from (5). The curves from the b-wires without cylinder, which is only $B_{zv}$, is seen in A to be about 5% of the uppermost curve, for $y < 5$ cm. For $y > 5$ cm the effect of the smaller tank used here becomes apparent; for this reason the approximation to a semi-infinite conductor breaks down. The ratio of the latter curve to the former curves is not quite as low as the gradient ratios of the second experiment; nevertheless, to within experimental error, this further evidence that $B_{zv} = 0$ is convincing. As with the flat insulator the effect of the cylindrical insulator, as shown in the lowest curves, is strong. The negative polarity confirms that these curves are due to the volume current, which flows in the opposite direction to the dipole current. The curves due to the a-wires with cylinder is seen to be the difference between the uppermost curve for $y < 5$ cm, as it should. Again, for $y > 5$ cm the tank boundary breaks down this relationship. These curves simulate the variation of MCG amplitude across the chest, when the heart can be approximated by a single dipole and the lung as a pure cylindrical insulator. It is noteworthy that the shape does not deviate from that due to a dipole element; this suggests that the actual MCG may not be a strong function of the volume current. Indeed computer simulation of the lungs in the torso, as described in the third paper in this group, shows that $B_{zv}$ due to the lung is much less than one would expect from Fig. 8 and the lower curves of Fig. 10. Presumably this is due to the low resistivity of the actual lung in comparison to the insulator used here. A comparison
between the upper curves of A and B shows that $B_z$ follows the inverse-square falloff from the electrodes, as they should according to (5).

Fig. 11. Values of $B_z$ and its derivatives as a function of $y$, due to a single dipole (solid lines), and to an opposing dipole pair (broken lines). These were computed by using (5) and the dimensions in A; although MKS units are used in text equations, results here are given in gauss and cm. The opposing pair is the basic combination of interest magnetically because it yields a suppressed potential in comparison to $B_z$. The ratio of the maximum surface potential of the pair to that of a dipole is 0.14, while the equivalent ratio is here shown to be 0.5; a factor of 3.5 is therefore the magnetic enhancement.

To magnetically detect the presence of the pair (inverse problem) a comparison among B, C, and D shows that $\partial^2 B_z/\partial z^2$ is the most sensitive quantity to use. The curve of $B_y$ is due to one dipole and is also computed from (5); it is shown here to indicate the narrower resolution of $\partial B_z/\partial y$, in comparison, due to differentiation. The potential and all quantities shown here increase linearly with the distance between the dipole pair.
These experiments, as a group, strongly confirm that \( B_{zv} = 0 \) for the semi-infinite volume conductor. The bases of the derivation and arguments given previously are therefore confirmed. We may conclude that \( B_{zv} = 0 \) for the other shapes mentioned, such as the infinite volume conductor and the sphere. All our measurements support this conclusion.

**DISCUSSION AND APPLICATION**

It is shown in the third paper in this group that, for the case of the inhomogeneous torso, \( B_{zv} \) is either small or can be estimated; hence, in many circumstances \( B_z \approx B_{z0} \) or \( B_{z0} \) can be estimated as a result of (1). Because of this, we can now understand the MCG falloff with distance from the chest. For each equivalent dipole current generator in the heart, \( B_z \) varies as \( z^{-2} \) due to (5). If the heart can be approximated by only one dipole, say, then the MCG would vary as \( z^{-2} \). This is borne out by the measured MCG variation with distance from the chest directly over a normal heart, where \( B_{zv} \) is often small and there is a large single-dipole component. Generally, this explains the variation of MCG amplitude with chest thickness, hence with the subject's dimensions and weight. Because of the inverse-square falloff, clearly a subject with a thick chest will produce a smaller MCG than a thin subject with an equivalent heart. Again, this is mostly due to the greater separation between the equivalent current dipole and the detector, not to any involvement of the return current. It is possible, based on this knowledge, to deduce an MCG index which would be the counterpart of the ECG ponderal index (although different). As an aside, we note a special result from the fact that a spherical homogeneous volume conductor is a good approximation to the human head; to an even better approximation than for the heart \( B_z \approx B_{z0} \); therefore for each equivalent dipole in the brain the normal component \( B_z \) will fall off as \( z^{-2} \) quite closely.

The fact that \( B_z \approx B_{z0} \) for the torso means that (5) can now be applied to calculate \( B_z \) and related quantities due to those combinations of dipoles which are of interest in the interpretation of the MCG. These are the combinations which produce a surface potential which is suppressed, but an external \( B_z \) which, relatively, is not suppressed. Stated otherwise, a major justification for MCG measurements is to identify those generator configurations which produce potentials which are too low to measure with the ECG. We know the equivalent dipole arrangements which do this, and (5) can now be used to simply solve the forward problem for these arrangements. The solutions would be the distribution of \( B_z \) and related quantities; these could be studied as aids in solving the inverse problem.

The most basic dipole combination for this purpose is an opposing pair. Solutions for this pair, as well as a single dipole, are shown in Fig. 11. The solution of the forward problem for \( B_z \) is shown in Fig. 11a, while related forms of the solutions are shown in C and D. Other calculations (not described here) show that the ratio of the maximum potential of the pair to that of a single dipole
is 0.14. It is seen in Fig. 11B that the same ratio for $B_z$ is 0.5. The magnetic gain over the potential is therefore a factor of about 3.5; this factor can be shown to be independent of the spacing between the two dipoles, for small spacing. A gain of 3.5 is not particularly impressive. However, it has been shown that this factor increases greatly as the number of dipoles in the configuration is increased to three or more "around a loop". These latter cases, we therefore believe, are only revealed by magnetic measurements and are a major justification for magnetic measurements.

We now ask: what are the most useful ways to extract information about the dipole pair from a measured distribution of $B_z$? The curves of Fig. 11B, C, and D yield one useful answer to this question concerning the inverse solution. While $B_z$ and $\partial B_z/\partial y$ for the pair are about one-half that of the single dipole, $\partial^2 B_z/\partial z^2$ is the same for pair and dipole; this second derivative, therefore, is twice as sensitive for the detection of the dipole pair as is $B_z$ or $\partial B_z/\partial y$. In a sense, the higher the spatial derivative of $B_z$ which is used, then the greater is the sensitivity to the higher spatial frequencies. In practice $B_z$ can be measured across the torso at several different spacings from the skin, and the second derivative can then be computed from these measurements. One guesses, however, that performing the differentiation directly within the detector (which is then called a second-derivative gradiometer) would yield less error of computation. Some measurements of this type have already been made.

The concept of dipole resolution is involved here. An instructive picture is the following. Consider $B_z$, $\partial B_z/\partial y$, and $\partial^2 B_z/\partial z^2$ due to one dipole; these are the solid curves in Fig. 11. We then bring up an opposing second dipole from a distant point, and consider the $y$-distribution of the three quantities as the distance decreases between the two. When the dipole is still some distance away, say 20 cm, all three quantities will show two distinct patterns, hence two resolved dipoles. As the second dipole is brought closer, in all three quantities the double pattern will begin to emerge as a single pattern, finally yielding the curves of Fig. 11. During this latter step, the spacing between the dipoles at which they can be just visually resolved is, by inspection, least for $\partial^2 B_z/\partial z^2$; this quantity therefore yields the highest resolution. While the eye cannot resolve the dipoles by inspection of the broken curves of Fig. 11, the information nevertheless exists in these curves; this information can be extracted by other means from these merged patterns, and will yield the presence and some parameters of the dipole.

There are some aspects of using the derivatives of $B_z$ which should be noted. The first of these is the more rapid falloff with distance of the derived quantity. The quantity $\partial^2 B_z/\partial z^2$ falls off as $z^{-4}$ from a current dipole, hence the second-derivative MCG strongly favors the anterior portion of the myocardium. Of course a second-derivative MCG measurement over the posterior torso surface would yield information concerning the posterior portion of the myocardium, hence there are both drawbacks and advantages to this rapid falloff. Stated in ECG terms, a measurement of $\partial^2 B_z/\partial z^2$ constitutes a highly aimed lead for detecting an opposing dipole pair. Another aspect is the decrease in the detector ratio of
signal/noise with increasing order of derivative of $B_z$. The internal instrumental noise of the superconducting SQUID detector is not insignificant. The detecting coil which feeds the SQUID, called the flux transformer, yields a much weaker signal with increasing differentiation, resulting in decreased signal/noise. A second-derivative gradiometer is, in effect, two opposing first-derivative gradiometers, so we are here taking the difference between two small quantities. The technical difficulties of recording the MCG increase, therefore, with the degree of differentiation used in the gradiometer. However, the state of the SQUID detection art is continuously improving and the internal noise is continuously being decreased. In time, it is believed, the noise problems of a second-derivative gradiometer will be solved.

We next discuss methods of displaying distributions of $B_z$ which either have been experimentally measured or have been computed. We refer especially to the MCG, in which $B_z$ is here the field component normal to the chest, although this discussion can also pertain to the magnetic field from the brain or other sources. Because it is impractical to record MCG’s across the chest with a spacing which is finer than about 5 cm, the well-known isopotential type of instantaneous map cannot readily be used. Instead we have been using a discrete type of map consisting of black and open squares for displaying instantaneous $B_z$ distributions. We here suggest an alternate form for displaying the MCG across the chest in which arrows are used. The same information is contained in the arrow map as with the square map. However, in the arrow map a field is presented differently to the eye, and some information is more readily determined visually in this way than with the square map. This information is the location and orientation of a single dipole, or of an opposing dipole pair and multiples thereof. The arrow map concept is tied to the property $B_z=B_{zd}$; if $B_{zv}$ is relatively large, then the arrow map may not be useful. We define each arrow to be the vector

$$\vec{a} = \left(\frac{\partial B_z}{\partial y}\right)\hat{x} - \left(\frac{\partial B_z}{\partial x}\right)\hat{y}$$

where $\hat{x}$ and $\hat{y}$ are unit vectors. This is the two-dimensional gradient of the scalar quantity $B_z$, but is rotated by $90^\circ$ from the more conventional gradient. This gradient vector was chosen because the arrows are largest immediately over the dipole and co-linear with it, at least when $B_z=B_{zd}$. The purpose of these arrows, therefore, is to directly indicate to the eye the direction and location of the underlying dipole when $B_z=B_{zd}$.

Comparisons between square and arrow maps are shown in Figs. 12, 13, and 14; further arrow illustrations are shown in Figs. 15 and 16. Figures 12, 13, and 14 contain the four basic dipole combinations which are used here. A single dipole is shown in A of these figures; the type of opposing dipole pair which is useful magnetically is shown in B; the reverse type of opposing pair is shown in C, while four around-a-loop is shown in D. Before comparing the square map and arrow map displays of $B_z$ (Figs. 13 and 14) it is useful, as an aside, to compare the potential map of Fig. 12 to the $B_z$ map of Fig. 13. These are both shown in square-map format because this format has usually been used by us in the display of $B_z$. 

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and the potential (no boundary) show similar distributions, except for the 90° rotation, discussed earlier. This 90° rotation explains B and C. It is seen that the two dipole configurations in B and C are interchangeable; that of B gives a large $B_z$ but small potentials, while that of C gives large potentials but small $B_z$. From this it can be ascertained that potential and $B_z$ measurements are complementary; one method brings out the dipole combination which is suppressed by the other. Two pairs of opposing dipoles as in B yield the four around-a-loop shown in D, hence this configuration is favored magnetically and highly suppressed potentially. This arrangement almost certainly is hidden to potential measurements, although readily detectable magnetically. In comparing Fig. 13 and 14, we see that the arrow map reveals the location of the dipoles in A, B, and D more effectively to the eye than does the square map. The dipoles are here 5 cm apart as compared with 2 cm in Fig. 11, for purposes of illustration. In Fig. 14D, the blank space within the four dipoles bears discussion. This blank space indicates zero gradient because of the resolution aspect of this type of calculation; we present some resolution data in Fig. 15. Figure 15A shows the result when the dipole is only 5 cm below the plane of calculation instead of 10 cm; in this case there is not much artifact or distortion. The same holds true for B, in which the dipoles of the opposing pair are moved closer together; the eye still can identify this pair easily from the arrow map. The distributions of C and D show opposing effects; C was computed for dipoles 5 cm below the surface while D was computed with the dipoles 10 cm below the surface, but far apart. A pronounced number of zero-gradient points is seen in C. However, D shows the effect of dipoles far apart, hence without the cancellation effects in the center. It is seen that the dipole arrangement is still easily identified visually, as compared with the square map. Figure 16 shows arrow maps due to other dipole arrangements. It is seen in this figure that some of these dipole arrangements, less basic than those of Fig. 15, can partially be determined visually as well. This holds true for A and D, while this is not true for B and C. One expects that for C, for example, the square map or isopotential type of display would be more favorable. The purpose of these combinations is to acquaint the eye with the type of patterns which might be seen on the MCG arrow map displays of $B_z$, hence as an aid to visual determination.

Any scalar field, magnetic or otherwise, can be displayed with similar arrows. With some types of scalar fields this gradient display can readily reveal to the eye some information concerning the generators of the field. This display does not create new information, and indeed has the same information content as other displays derived from the same field. It is only a visual aid which seems well-suited for the display of $B_z$ fields. Partly this is because it shows an opposing pair clearly, and this is an arrangement we tend to seek by magnetic measurements. Arrow maps do not seem to be as useful for the display of the potential due to the heart, that is, as a substitute for the isopotential map. This is because a large number of potential points can be readily sampled so that isopotential lines can be used; also, combinations of dipoles other than the opposing pair are of interest in potential determination. It appears that some attempts have
been made by one or more authors to use a form of arrow map for the heart's potential, although for technical reasons and in a form not fully defined.

In the third and fourth sections of this report we use arrow maps for displaying the instantaneous MCG's in order to aid in the visual determination of the cardiac generators which produced these MCG's.
Fig. 12. Potential distributions in a plane (no actual boundary) due to underlying charge dipoles. The square map display is here used for comparison with the \( B_y \) square map of Fig. 13. Dipole parameters are identical to those of Fig. 13. Dipoles are 10 cm below plane of calculation and 5 cm apart; computed points are 5 cm apart. The length of the side of a square is proportional to the potential; a solid square denotes positive potential and an open square denotes negative potential. Whereas the dipoles in B yield low potentials and those in C yield high potentials, the situation is reversed magnetically (see Fig. 13) due to a 90° rotation difference between potential and field.
Fig. 13. Distributions of $B_\parallel$ computed from equation (5), using the square map display. The length of the side of a square is proportional to $B_\parallel$; solid square denotes positive polarity, open square denotes negative polarity. Current dipoles, shown as dark arrows, are 10 cm below plane of calculation and are 5 cm apart; squares are also 5 cm apart. The dipole arrangement shown in C is suppressed magnetically, but the potentials are not suppressed. This map is to be compared both to the potential map of Fig. 12, and to the arrow map of Fig. 14.
Fig. 14. Arrow map display of same distributions of $B_z$ as shown in Fig. 13. Each arrow is the gradient of $B_z$ rotated by $90^\circ$, and is expressed by

$$\mathbf{a} = \left(\frac{\partial B_z}{\partial y}\right)\mathbf{\hat{x}} - \left(\frac{\partial B_z}{\partial x}\right)\mathbf{\hat{y}}.$$  

The scales are here arbitrary, but all the dipole strengths are identical. The current dipoles are again separated by 5 cm, and the arrows by 2.5 cm. The open space within the four dipoles of C indicates zero gradient; see Fig. 150 for different parameters of this arrangement. It is seen that the underlying dipoles are easier to locate in the arrow map than in the square map, except for the pair in C.
Fig. 16. Resolution effects on arrow maps due to altered spacing of current dipoles. As in Fig. 14, the arrows are separated by 2.5 cm in the calculation. In A the dipole is 5 cm below the surface, instead of 10. In B the dipoles are 1.25 cm apart instead of 5 cm, and 10 cm below the plane of computation. In C the dipoles are 5 cm below the plane of computation, and 5 cm apart. In D the dipoles are again 10 cm below the plane, but are now 15 cm apart. The dipole strengths are not identical here, but have been adjusted to yield convenient arrow lengths in each of the four cases.
Fig. 18. Arrow maps due to other arrangements of current dipoles. Dipoles are always 10 cm below plane of calculation with a spacing as indicated; arrows are 2.5 cm apart. In A and D the arrangement of underlying dipoles can be readily determined visually, while in B and C the visual determination is poor. The dipole strengths are proportional to the lengths of the solid arrows (in this figure only). The "magnifications" are identical in A, B and D, but is a factor of three in C. The gradients in C are therefore very small in comparison to A, B, and D.
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12. Cohen, D., Hosaka, H., Lepeschkin, E., Levine, H.D., Massell, B.F., McCaughran, D., and Myers, G.: Magnetocardiograms of Normal and Abnormal Subjects. This is the first of four sections of this report. Reference 3 (above) is an abridged version of this particular section; all four abridged sections have been accepted for publication in the J. of Electrocardiology.

