DIFFUSE PHOTON PROBES OF STRUCTURAL AND DYNAMICAL PROPERTIES OF TURBID MEDIA: THEORY AND BIOMEDICAL APPLICATIONS

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ABSTRACT

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Diffusing photons can be used to detect, localize, and characterize optical and dynamical spatial inhomogeneities embedded in turbid media. Measurements of the intensity of diffuse photons reveal information about the optical properties of a system. Speckle fluctuations carry information about the dynamical and optical properties. This dissertation shows that simple diffusion theories accurately model the intensity and speckle correlation signals that diffuse through turbid media with spatially varying properties and discusses possible biomedical applications.

We first look at the intensity of diffuse photons provided by a light source that is intensity modulated. This generates diffuse photon density waves (DPDW's) which exhibit classical wave behavior. We demonstrate experimentally and theoretically the refraction, diffraction, and scattering of DPDW's. Using accurate signal and noise models, we then present a detailed analysis which shows that DPDW's can be used to detect and locate objects larger than 3 mm and to characterize objects larger than 1 cm which are embedded inside turbid media with biologically relevant parameters. This diffuse photon probe should may find applications in medicine as a bed-side brain hematoma monitor, or for screening breast cancer, or other functional imaging applications.

We then consider the coherence properties of the diffuse photons as revealed by speckle intensity fluctuations and show that the temporal autocorrelation function of these fluctuations is accurately modeled by a correlation diffusion equation. Because the correlation diffusion equation is analogous to the photon diffusion equation, all concepts and ideas developed for DPDW's can be directly applied to the diffusion of correlation. We show experimentally and with Monte Carlo simulations that the diffusion of correlation can be viewed as a correlation wave that propagates spherically outwards from the source and scatters from macroscopic spatial variations in dynamical and/or optical properties. We also demonstrate the utility of inverse scattering algorithms for reconstructing images of the spatially varying dynamical properties of turbid media. The biomedical applicability of this diffuse probe is illustrated with examples of monitoring blood flow and probing the depth of burned tissue.

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