# Model-Based Reconstruction for Joint Estimation of $T_1$ , $T_2^*$ and $B_0$ Inhomogeneity Maps Using Single-Shot Inversion-Recovery Multi-Echo Radial FLASH

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## Introduction

Inversion-recovery (IR) Look-Locker and multi-echo gradient-echo are two commonly-used sequences for  $T_1$  mapping, and water-fat,  $T_2^*$  imaging, respectively [1-6]. Recently, these two sequences have been combined in one single scan for simultaneous multi-parameter mapping [6-9], providing complementary quantitative information for clinical studies. To obtain robust parameter estimation, most existing approaches usually consist of several steps, e.g.,  $B_0$  calibration/estimation, linear image reconstruction, and pixel-wise fitting. As each step utilizes a subset of acquired data, existing approaches may have not made the best use of all available data.

In this work, we propose to jointly estimate  $T_1$ ,  $T_2^*$  and  $B_0$  maps directly from the k-space data acquired from a single-shot IR radial multi-echo FLASH by formulating parameter estimation as a nonlinear inverse problem [10-11]. In this way, all acquired data can be exploited for a joint reconstruction. Furthermore, prior knowledge such as joint sparsity and  $B_0$  smoothness can be applied directly to parameter maps to improve the conditioning of the inverse problem.

### Methods

### Sequence Design and Model-based Reconstruction

The IR multi-echo sequence is shown in Fig.1. It starts with a non-selective inversion pulse, followed by a continuous radial multi-echo spoiled gradient-echo (FLASH) using the small golden-angle readout. The signal evolution for this process can be described as:

$$S_{\mathrm{TI}_{n},\mathrm{TE}_{m}} = \begin{bmatrix} W_{ss} - (W_{ss} + W_{0}) \cdot \exp\left(-\mathrm{TI}_{n} \cdot R_{1,W}^{*}\right) \\ + \left(F_{ss} - (F_{ss} + F_{0}) \cdot \exp\left(-\mathrm{TI}_{n} \cdot R_{1,F}^{*}\right)\right) \cdot z_{m} \end{bmatrix} \\ \cdot \exp\left(\mathrm{TE}_{m} \cdot i2\pi f_{B_{0}}\right) \cdot \exp\left(-\mathrm{TE}_{m} \cdot R_{2}^{*}\right).$$
(1)

Where  $(W_{ss}, W_0, R_{1,W}^*)^T$  are the steady-state signal, the equilibrium-state signal and the effective  $T_1$  relaxation rate for water and  $(F_{ss}, F_0, R_{1,F}^*)^T$  represent the corresponding signal components for fat.  $z_m$  is the 6-peak fat spectrum,  $f_{B_0}$  represents the field inhomogeneity and  $R_2^*$  is the  $T_2^*$  relaxation rate.  $\text{TI}_n$  and  $\text{TE}_m$  denote the *n*-th inversion time and *m*-th echo time, respectively. The unknowns optimized for are  $\boldsymbol{x} = (W_{ss}, W_0, R_{1,W}^*, F_{ss}, F_0, R_{1,F}^*, f_{B_0}, R_2^*)^T$ . Their estimation is formulated as a nonlinear inverse problem:

$$\boldsymbol{x} = \operatorname{argmin}_{\boldsymbol{x} \in D} \sum_{\mathrm{TI}} \sum_{\mathrm{TE}} \|P\mathcal{F}C \cdot S_{\mathrm{TI}_{n},\mathrm{TE}_{m}}(\boldsymbol{x}) - Y_{\mathrm{TI}_{n},\mathrm{TE}_{m}}\|_{2}^{2} + R(\boldsymbol{x}).$$
(2)

Here D is a convex set, ensuring non-negativity of all relaxation rates and  $R(\boldsymbol{x})$  is a regularization term. We adopt a joint l1-Wavelet sparsity constraints [11] on all parameters of  $\boldsymbol{x}$  except  $f_{B_0}$ . The latter is regularized with a smoothness enforcing Sobolev penalty [12] also added to the coil sensitivities. The nonlinear inverse problem in Eq.2 is solved with a IRGNM-FISTA [11] using BART [13].

# Experiments

All MRI studies were conducted on a 3T scanner (Magnetom Skyra, Siemens Healthineers, Erlangen, Germany) with approval of the local ethics committee. The brain study was conducted using a 20-channel head/neck coil. The acquisition parameters were: FOV=192 × 192 mm<sup>2</sup>, matrix size=256 × 256, slice thickness = 5 mm, 7 echos with TR=15.6 ms TE<sub>1-7</sub>=2.36/4.26/6.16/8.06/9.96/11.90/13.80 ms, FA=6°, bandwidth=810 Hz/pixel and 450 excitations in total with 3350 radial acquired spokes. Nice spokes were combined into one k-space frame for fast computation.

### **Results & Discussion**

The proposed model-based reconstruction was first validated for a numerical phantom, which provides ground truth in the presence of noise. Fig.2(A) presents the determined quantitative water  $T_1$ , fat  $T_1$ ,  $T_2^*$  and  $f_{B_0}$  maps from the model-based reconstruction. Fig.2(B) shows the corresponding Bland-Altman plots comparing quantitative values estimated from ROIs to the known ground truth. The small differences observed in the Bland-Altman plots confirm good accuracy for all parameter maps.

Fig.3 further demonstrates model-based reconstructed brain water  $T_1$ , fat  $T_1$ ,  $T_2^*$  and  $f_{B_0}$  maps. The steady-state water and fat images (i.e.,  $W_{ss}$  and  $F_{ss}$  in Eq.1) are additionally shown in the second row. Although a reference method need to be performed for a pairwise comparison, visual inspection shows artifact-free quantitative  $T_1$  and  $T_2^*$  maps. The quantitative values estimated from ROIs ( $T_1$ : white matter:  $800\pm 8$  ms gray matter:  $1388\pm 61$  ms;  $T_2^*$ : white matter:  $46\pm 2$  ms gray matter:  $61\pm 3$  ms) correspond well to the literature values [4,10].

## Summary

The presented work formulates the joint estimation of  $T_1$ ,  $T_2^*$  and  $f_{B_0}$  maps from a single-shot IR multi-echo acquisition as a single nonlinear inverse problem. Initial results on a simulated phantom and a healthy subject demonstrate that high resolution and artifact-free quantitative maps can be reconstructed simultaneously with the proposed method.

#### References

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This work was partially funded by NIH under grant U24EB029240. We thank Dr. Li Feng for insightful discussions.

Figure 1: Schematic diagram of the inversionrecovery multi-echo FLASH sequence and its asymmetric radial sampling pattern.



Figure 2: (A) Model-based reconstructed quantitative water- $T_1$ , fat- $T_1$ ,  $T_2^*$  and  $fB_0$ maps from a single-shot IR multi-echo radial FLASH acquisition for a simulated phantom. (B) Bland-Altman plots comparing ROI-analyzed mean quantitative values to the ground truth.



Figure 3: (Top). Model-based reconstructed artifact-free brain water  $T_1$ , fat  $T_1$ ,  $T_2^*$  and  $fB_0$  maps from a single-shot IR multi-echo radial FLASH acquisition. (Bottom). Steadystate water and fat images (i.e.,  $W_{ss}$  and  $F_{ss}$ in Eq.1) from the model-based reconstruction.