Intensity-based registration

Course 22525

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Course structure

Fitting functions

Registration

Segmentation
Image registration

Determine a geometrical transformation that aligns points in an image with corresponding points in other image(s)
Image registration

Determine a geometrical transformation that aligns points in an image with corresponding points in other image(s)
Elements in image registration

- Geometrical transformation $y(x; w) : \mathbb{R}^2 \to \mathbb{R}^2$ or $\mathbb{R}^3 \to \mathbb{R}^3$
- Similarity measure $\mathcal{D}(w)$
- Regularization $\mathcal{S}(w)$
- Optimization algorithm $\mathcal{J}(w) = \mathcal{D}(w) + \alpha \mathcal{S}(w)$
Elements in image registration

- Geometrical transformation $y(x; w) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ or $\mathbb{R}^3 \rightarrow \mathbb{R}^3$

- Similarity measure $\mathcal{D}(w)$

- Regularization $\mathcal{S}(w)$

- Optimization algorithm $\mathcal{J}(w) = \mathcal{D}(w) + \alpha \mathcal{S}(w)$
Transformations (linear)

**Translation**

\[ y(x; t) = x + t \]

**Rigid: Translation + Rotation**

\[ y(x; R, t) = Rx + t \]

\[ R^T R = I, \quad \det(R) = 1 \]

**Similarity Transformation**

\[ y(x; s, R, t) = sRx + t \]

**Affine Transformation**

\[ y(x; A, t) = Ax + t \]

\[ s > 0 \]
Transformations (non-linear)
Landmark based registration

- Geometrical transformation \( y(x; w) : \mathbb{R}^2 \to \mathbb{R}^2 \) or \( \mathbb{R}^3 \to \mathbb{R}^3 \)
- Similarity measure \( \mathcal{D}(w) \)
- Regularization \( \mathcal{S}(w) \)
- Optimization algorithm \( \mathcal{J}(w) = \mathcal{D}(w) + \alpha \mathcal{S}(w) \)

Match two corresponding point sets defined in two images: \( x_i, y_i \in \mathbb{R}^2 \) or \( \mathbb{R}^3 \)

\[
\mathcal{D}(w) = \sum_{i=1}^{N} \| y(x_i; w) - y_i \|^2
\]
Rigid landmark based registration
Landmark based registration

- Geometrical transformation $y(x; w) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ or $\mathbb{R}^3 \rightarrow \mathbb{R}^3$
- Similarity measure $D(w)$
- Regularization $S(w)$
- Optimization algorithm $J(w) = D(w) + \alpha S(w)$

I'm too lazy to put landmarks. Can't we use images directly?
Intra-modality registration
Intra-modality registration
Intra-modality registration

- Sum of squared differences (SSD)

\[ D_{SSD}(w) = \frac{1}{2} \sum_{i \in \Omega} (T(y(x_i; w)) - R(x_i))^2 \]
Intra-modality registration

- Sum of squared differences (SSD)

\[
D_{\text{SSD}}(w) = \frac{1}{2} \sum_{i \in \Omega} \left( T(y(x_i; w)) - R(x_i) \right)^2
\]
Intra-modality registration

• Sum of squared differences (SSD)

\[ D_{\text{SSD}}(w) = \frac{1}{2} \sum_{i \in \Omega} (T(y(x_i; w)) - R(x_i))^2 \]
Intra-modality registration
Miscalibration of intensities
Miscalibration of intensities
Miscalibration of intensities

Mean: 73.36
Std. dev.: 59.49

Mean: 129.58
Std. dev.: 24.45
Miscalibration of intensities

• Subtract mean
Miscalibration of intensities

- Divide by standard deviation
Miscalibration of intensities

• One more thing:

\[ \sum_i (r_i - t_i)^2 = \sum_i (r_i^2 - 2r_it_i + t_i^2) = \sum_i r_i^2 + \sum_i t_i^2 - 2 \sum_i r_it_i \]
Miscalibration of intensities

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Miscalibration of intensities

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Miscalibration of intensities

- Correlation coefficient (CC)

\[
D_{CC}(w) = \frac{\sum_{i \in \Omega} (T(y(x_i; w)) - \bar{T}) (R(x_i) - \bar{R})}{\sqrt{\sum_{i \in \Omega} (T(y(x_i; w)) - \bar{T})^2} \sum_{i \in \Omega} (R(x_i) - \bar{R})^2}
\]

\[
\bar{T} = \frac{1}{|\Omega|} \sum_{i \in \Omega} T(y(x_i; w))
\]

\[
\bar{R} = \frac{1}{|\Omega|} \sum_{i \in \Omega} R(x_i).
\]
Inter-modality registration
Inter-modality registration
Inter-modality registration
Inter-modality registration
Inter-modality registration

Idea: build a joint histogram
Inter-modality registration

Idea: build a joint histogram
Inter-modality registration

Idea: build a joint histogram
Inter-modality registration

MR intensities

CT intensities
Inter-modality registration

CT intensities

MR intensities
Inter-modality registration

- Entropy

- Also works for more than two outcomes
Inter-modality registration

• Joint entropy

\[ D_{JE}(w) = H(\mathcal{T}(y(x_i; w)), R(x_i)) \]

\[ H(\mathcal{T}(y(x_i; w)), R(x_i)) = - \sum_{j,k} \text{PDF}(j, k) \log(\text{PDF}(j, k)) \]

\[ \text{PDF}(j, k) = \frac{\text{HIST}(j, k)}{\sum_{j,k} \text{HIST}(j, k)} \]
Inter-modality registration

- Wait a minute...
Inter-modality registration

• Mutual information

\[ D_{\text{MI}} = H(\mathcal{T}(y(x_i; w)), \mathcal{R}(x_i)) - H(\mathcal{T}(y(x_i; w))) - H(\mathcal{R}(x_i)) \]
Inter-modality registration
Inter-modality registration
Initialization?
Principal axes transform

\[
\mu_R = \frac{\sum_{\Omega_R} \mathcal{R}(x_i)x_i}{\sum_{\Omega_R} \mathcal{R}(x_i)}
\]

\[
\Sigma_R = \frac{\sum_{\Omega_R} \mathcal{R}(x_i)(x_i - \mu_R)(x_i - \mu_R)^T}{\sum_{\Omega_R} \mathcal{R}(x_i)}
\]

\[
\mu_T = \frac{\sum_{\Omega_T} \mathcal{T}(y_i)y_i}{\sum_{\Omega_T} \mathcal{T}(y_i)}
\]

\[
\Sigma_T = \frac{\sum_{\Omega_T} \mathcal{T}(y_i)(y_i - \mu_T)(y_i - \mu_T)^T}{\sum_{\Omega_T} \mathcal{T}(y_i)}
\]